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AUTHOR Serdahl, Eric
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ABSTRACT

The information that is gained through various analyses of the residual scores yielded by the least squares regression model is explored. In fact, the most widely used methods for detecting data that do not fit this model are based on an analysis of residual scores. First, graphical methods of residual analysis are discussed, followed by a review of several quantitative approaches. Only the more widely used approaches are discussed. Example data sets are analyzed through the use of the Statistical Package for the Social Sciences (personal computer version) to illustrate the various strengths and weaknesses of these approaches and to demonstrate the necessity of using a variety of techniques in combination to detect outliers. The underlying premise for using these techniques is that the researcher needs to make sure that conclusions based on the data are not solely dependent on one or two extreme observations. Once an outlier is detected, the researcher must examine the data point's source of aberration. (Contains 3 figures, 5 tables, and 14 references.) (SLD)

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An Introduction to Graphical Analysis of Residual Scores and Outlier Detection
in Bivariate Least Squares Regression Analysis

Eric Serdahl

Texas A & M University 77843-4225

Paper presented at the annual meeting of the Southwest Educational Research Association,

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Regression analysis can be defined as an analysis of the relationships among variables. In least squares linear regression the goal is to establish an equation that represents the optimal linear relationship between the observed variables. This relationship is represented by the equation

$$Y_i = a + bX_i + e_i,$$

where a and b are the estimated intercept and slope parameters, respectively and e_i represents the error in estimating Y_i . The regression equation yielded by the least squares approach is an equation for the predicted values of Y_i , not the actual values of Y_i :

$$\hat{Y}_i = a + bX_i,$$

where $\hat{Y}_i$ is the unobserved, predicted value of Y_i that falls on the regression line for each observation of Y . Hence, Y minus $\hat{Y}$ for each observed value will equal the aforementioned error term, e . This error term is commonly referred to as the residual score.

The residual score for an observation is the distance in units of Y between the observed data point and the line defined by the regression equation. In the regression model Y is a linear function of X and the residual score is a measure of the discrepancy in that approximation. It follows then that the closer to zero the residual scores are, the more accurately the predicted $\hat{Y}$ values will reflect the empirical Y scores. That is, as the residuals get smaller a stronger linear relationship is suggested between the dependent and independent variables. The least squared regression approach is designed to minimize the sum of the residual values while *assuming* a linear relationship among the variables being studying. Finally, it should be noted that regression analysis involving more than one independent variable generates a "plane" or "hyperplane" as opposed to a "line" of best fit, however, the basic calculations regarding the residual scores are the same.

The present paper will deal with the information that is gained through various analysis of the residual scores yielded by the least squares regression model. In fact, the most widely used methods for detecting data that do not fit this model are based on an analysis of residual scores (Rousseeuw & Leroy, 1987). First, graphical methods of residual analysis are discussed followed by a review of several quantitative approaches. Only the more widely used approaches will be discussed here, as there are many types of analysis that have been developed (and are in development) to identify the existence of problem data that attenuate the descriptive statistics generated by the regression model (Hecht, 1992). Example data sets are analyzed through the use of SPSS/PC to illustrate the various strengths and weaknesses of these approaches and to demonstrate the necessity of using a variety of techniques in combination to detect outliers.

Underlying Assumptions in Regression

The accuracy of the regression model in terms of explaining relationships among variables is based on a set of assumptions regarding the population from which the data are obtained. Before reviewing techniques that ensure the data reasonably match the regression model, the assumptions underlying the model are briefly reviewed. As outlined by Glantz and Slinker (1990) the assumptions are: (a) The relationship between the variables is linear, that is, the regression line passing through the data must do a "reasonably" good job of capturing the changes in Y that are associated with a change in X for all of the data; (b) for any given values of the independent variables, the possible values of the dependent variable are distributed normally; (c) the standard deviation of the dependent variable about its mean at any given values of the independent variables is the same for all values of the independent variables. Moreover, the spread about the best-fitting line in a scattergram must be about the same at all levels of the two variables, this is known as homoscedasticity; (d) the deviations of all members of the population from the best

fitting line or plane of means (as is the case in regression analysis involving more than one independent variable) are statistically independent. That is, a deviation associated with one observation has no effect on the deviations of other observations.

When the data do not fit these assumptions there is either erroneous data or the regression equation is inadequate in terms of describing the relationship between the variables. These two types of errors, known respectively as measurement error and specification error, can produce spurious interpretations based on regression analysis (Bohrnstedt & Carter, 1971). Therefore, it is beneficial to detect violations of these assumptions when doing regression analysis which can be achieved through careful examination of the residual scores. If the above assumptions are met then the residual e scores will be normally distributed about a mean of zero, homoscedastic, and independent of each other.

Graphical Analysis of Residuals

The following discussion is based in large part on the four scatter plots presented in Figure 1. These concepts were first illustrated by Anscombe (1973) in a seminal paper exploring the use of graphs in statistical analysis. Similar scatter plots have since been used in many text books on regression analysis to elegantly illustrate how a graphical analysis of residual scores can uncover hidden structures in the data that violate the least squares regression model (see Chatterjee & Price, 1991; Glantz & Slinker, 1990; Hamilton, 1990; Pedhazur, 1993).

INSERT FIGURE 1 ABOUT HERE.

The scatter plots in Figure 1 were created by plotting the residual e scores on the ordinate against the independent variable X values on the abscissas for fictitious data sets. As demonstrated by Anscombe (1973), data sets for all four graphs in Figure 1 can be created that

yield equivalent statistics through regression analysis. That is, data sets represented by these four graphs can be created that produce identical regression coefficients, equations for regression lines, regression sum of squares, residual sum of squares, and multiple R^2 . If all the observations are considered to be reliable without evidence of some gross recording error then the statistics would lead to similar interpretations, yet the data are remarkably different. This information would be undetected if such scatterplots were not examined.

If the assumptions underlying regression analysis hold, then the residual e scores will be normally distributed about a mean of zero with constant variance throughout the line originating through the mean of the residuals. In plainer language this can be stated as the e scores falling along a constant width band centered on zero in the residual graph. Figure 1A illustrates such a plot. That is, the residuals are reasonably randomly scattered about zero with constant variance regardless of the value of the independent variable, X . Further, none of the residuals is notably large in either the positive or negative direction, suggesting that there are no extreme "outlier" data points that disproportionally bias the linear regression equation (the concept of outliers will be more thoroughly discussed later in the paper). If the linear regression model was misspecified (meaning that the aforementioned underlying assumptions were violated), then the residuals would deviate from the expected pattern about zero. Trends up or down, curves, or a "megaphone" shape with one end of the band narrower than another in the residual plot would indicate violations of the homoscedasticity and/or linearity assumptions.

Figures 1B, C, and D represent residual patterns that call into question the validity of the regression analysis. In Figure 1B the pattern obviously violates the assumption that the variables are related linearly. Rather, there appears to be a curvilinear relationship with negative residuals concentrated at low and high values of X and positive residuals concentrated at intermediate

values of X . A quadratic rather than a linear equation would do a better job of describing the relationship between variables in this case. Barringer (1995) explains the procedures for analyzing such data. What is most notable about Figure 1C is the outlying data point associated with a large residual value. This particular outlier is known as an outlier in the Y direction and will greatly influence the slope and intercept of the regression equation. This is particularly true in view of the fact that all of the residuals except the outlier are systematically (not randomly) scattered about zero with positive residuals at low values of X and negative residuals at high values of X .

Figure 1D denotes another outlying data point that exerts great influence on the regression equation. However, in this case the data point is an outlier in the x direction and does not possess a large residual score (in fact, the residual score is 0). These data violate the assumption of homoscedasticity as they do not vary constantly about the line $e = 0$ and call into question the normality of the X variable. Further, the data set is simply unsuitable for linear fitting as the regression line is determined essentially by only one data point. Moreover, if the outlier were to be dropped from the analysis, the regression relation would be near zero. This can be more readily seen by examining a simple scatter plot of the dependent Y variable against the independent X variable for this data, as seen in Figure 2. If the data point to the far right were deleted or moved in any direction the regression equation would correspondingly be adjusted as the regression line would be tilted. As stated by Anscombe (1973), "We are usually happier about asserting a regression relation if the relation is still apparent after a few observations (any ones) have been deleted - that is, we are happier if the regression relation seems to permeate all the observations and does not derive largely from one or two" (p. 19).

INSERT FIGURE 2 ABOUT HERE.

In the case of multiple independent variables, residuals can be plotted against each independent variable and examined for model violations, as shown above. Thus, providing two dimensional graphs which are more easily comprehended at a glance than multidimensional graphs involving planes. Further, residuals can be plotted against the fitted $\hat{Y}$ values in either bivariate or multivariate regression. By standardizing the e and the $\hat{Y}$ scores so that the scales for both axis are the same in this plot, one can at a glance observe the relative dispersions of the fitted values and the residual scores. The researcher will obviously hope that the $\hat{Y}$ scores are near the observed Y scores so that the variability of the $\hat{Y}$ is greater than that of the residual e scores. This discussion is not an exhaustive listing of the variety of ways that scatter plots can be used in data analysis. It is hoped that this brief examination of the topic provides support for the common recommendation found in many statistical texts: *examine the plotted data*.

Four Quantitative Approaches to Residual Analysis and Outlier Detection

Four commonly used quantitative approaches to outlier detection are briefly reviewed and then used to examine fictitious data sets. The four methods are: standardized residuals, Leverage, studentized residuals, and Cook's distance. As alluded to above, outliers are data points that fall a considerable distance away from the majority of the other data. Outliers can be thought of as falling along two scales, namely along the X and Y scales. Further, there can be outliers in varying degree in both directions. An outlier in the Y direction will have a large residual e score associated with it and can potentially influence the regression parameters (i.e., slope and intercept) by "pulling" the regression line towards the score's Cartesian coordinate so as to minimize the residual e scores. An outlier in the X direction can also greatly influence the regression parameters, however, its residual score may be small as demonstrated in Figure 1D. Outliers in the direction of the independent variable are by no means uncommon, particularly in the social

sciences where the independent X variables are observed quantities subject to random variability. In fact, having outliers in relation to the independent variables is probably more likely when there is only one dependent variable and multiple independent variables, thus providing more opportunity for things to go wrong (Rousseeuw & Leroy, 1987).

Standardized Residuals: The standardized residual is one method of identifying outliers in the Y direction. If a regression analysis were carried out on the data represented in Figure 1C after deleting the outlier, considerably different regression parameters (e.g., R and weights) would be calculated. Such an outlier is said to have "influence" on the regression equation and can be identified by examining standardized residuals. As defined earlier, raw e scores are calculated as:

$$e_i = Y_i - \hat{Y}_i$$

Standardized residuals are derived by dividing the raw e scores by the standard deviation of the residual scores, also referred to as the standard error of estimate. The standard error of estimate can be thought of as an estimate of the standard deviation of the population about the line (or plane) of means. Thus the standardized residual is calculated by

$$e_s = e / s_{y \cdot x}$$

where e_s is the standardized residual, e is the raw residual, and $s_{y \cdot x}$ is the standard deviation of the residual scores. Because e and $s_{y \cdot x}$ are in the same units, e_s is unitless and does not depend on the scale of measurement used to quantify the dependent variable Y. The advantage of this is that now general rules of thumb can be established for how far away a data point must lie before it is identified as an outlier. A standardized residual score of 0 means that a data point falls on the regression line. Further, a standardized residual of 1 means the point falls 1 standard deviation from the regression plane or line. Likewise, a standard residual of 2 indicates that the data point is 2 standard deviations away and so on.

As noted earlier, one of the assumptions of regression analysis is that the population members are normally distributed about the plane of means. Hence, the residuals are normally distributed about the regression line or plane. Normal distribution by definition means that about two thirds of the data points fall within 1 standard deviation of the of the regression line and approximately 95% fall within 2 standard deviations. Moreover, one would expect about 66% of the standardized residuals to have absolute values below 1 and almost all of the standardized residuals to have absolute values below 2. A general rule of thumb then is to identify data points with standardized residuals of 2 or more as potential outliers. These are thought to be outliers that can potentially have great influence on the regression parameters.

Two caveats should be observed when analyzing standardized residual scores. First, a standardized residual does not mean a data point is necessarily a mistake in recording or a special outlying observation for which the hypothesis being studied does not hold. In all normal populations, about 5% of the members exceed 2 standard deviations from the mean. Therefore, the above rule of thumb should not be used rotely. Second, an analysis of standardized residual scores will not identify outliers in the X direction. The outlier in Figure 1D, as noted previously, has a residual score of 0 (hence, a standard residual score of 0) and would be missed by solely an analysis of the standardized residuals.

Leverage: A point that is an outlier in the x direction is called a potential leverage point. Because a leverage point falls far away from the center of the independent variable values it can exert great influence on the regression equation as seen in Figure 2. It would be helpful then to quantify the leverage value of any particular data point in order to identify such an influential point. Such a quantity in the case of one independent variable is given by the equation

$$h_i = 1/n + (X_i - \bar{X})^2 / \sum (X_i - \bar{X})^2,$$

where h_i is a measure of how much the observed value of the dependent variable affects the estimated $\hat{Y}$ value for the i th data point. In other words h_i is a measure of the relationship that transforms the observed Y_i values to the predicted $\hat{Y}_{\text{hat}_i}$ values for each individual data point. This transformation is analogous to the slope and intercept parameters used in regression that transform Y to $\hat{Y}$, however, these parameters are constants while h_i varies from point to point. Thus h_i can be used to diagnose individual data points that potentially are influential leverage points. Note that h_i depends only on sample size and the values of the independent variables. As the X_i values move farther away from the mean of the independent X values, the leverage increases. Hence, leverage is focused on identifying outliers in the x direction.

It would be desirable that all the points have the same influence on the predicted $\hat{Y}$ values so that all the leverage values would be equal and small. Moreover, if all the data points are contributing equally to the regression equation then a stronger case can be made for the predicted linear relationship. It can be shown that leverage values fall between 0 and 1 (Belsley & Welsch, 1980). It can also be shown that the expected (average) value of leverage is

$$\text{expected } h_i = (k + 1)/n$$

where k is the number of independent variables in the regression analysis. It has been suggested that the cut off point for identifying a potential leverage point is twice the expected h_i (Rousseeuw & Leroy, 1987). Therefore, when $h_i > 2(k + 1)/n$ for the point in question, it's designated as having high leverage and warrants further investigation. However, just because a data point has a high h_i value does not mean the point is necessarily exerting disproportional influence on the regression equation. For example, an outlier in the x direction may be identified while the regression analysis with or without the identified point generates similar results. This may happen where a cluster of data points produces a regression line that also runs through the

outlying point. Such a point strengthens our confidence in the regression equation by suggesting that a linear relationship holds true for extremely low or high values of the independent variable.

Studentized Residuals: Standardized residuals are normalized by dividing e by the standard error of measurement, s_{e_x} . If the data fit the assumptions underlying regression then s_{e_x} will be constant for all values of the independent variable, X . However, data points that lie further away from the "center" (center being defined by the mean of X and the mean of Y) of the data will obviously have greater error of measurement associated with them. Hence, one can be more confident about the location of the regression line for those points that cluster around the "center" of the data. It is possible then to take this effect into account when normalizing the residuals and increase the sensitivity of the residuals to outliers. This is accomplished by dividing each e_i value by its specific standard error, which is a function of how far X is from the mean of X . As noted earlier, this distance can be quantified by the leverage calculation. The equation for the studentized residual is

$$r_i = e_i / s_{e_x} \sqrt{1 - h_i}$$

The studentized residual is more sensitive to outliers than is the standardized residual because it is sensitive to outliers in both the Y and to some degree in the X direction. As can be seen in the equation, larger r_i values will be created by not only higher magnitudes of the e scores but also by higher magnitudes of leverage quantities.

The above equation represents what is typically referred to as the internally studentized residual because the estimate of s_{e_x} is computed by using all the data. An alternate calculation can be done by computing the standard deviation error after deleting the data point in question. This is typically referred to as the externally studentized residual. The reasoning is that robust regression results would not be effected by deleting one point. If the point is in fact an outlier

then the estimation of $s_{y \cdot x}$ would be inflated and so understate the magnitude of the outlier effect as $s_{y \cdot x}$ appears in the denominator of the equation. Removing the data point in question from the analysis makes the externally studentized residual more sensitive to outliers. Similar cut off points are recommended for identification of outliers for this technique as were recommended for the standardized residuals (i.e., studentized scores of 2 or more). However, instead of rotely applying the cutoff points, it is more informative to look at the relative differences among the scores.

Cook's Distance: So far the techniques reviewed have been directed at identifying "potential" influential outliers. The basic logic utilized by these techniques is that the further a point falls away from the center of the data, the higher the probability is that the point will influence the regression equation. However, as was mentioned in regards to leverage, a point can be identified by these techniques and not necessarily be disproportionally influential in regression analysis. A technique that more directly measures the actual influence of a data point is Cook's distance. This is accomplished by computing how much the regression coefficients change when the point in question is deleted. Cook's distance is given by

$$CD_i = (r_i^2 / k + 1) (h_i / 1 - h_i)$$

CD_i can be thought of as a distance between one set of regression coefficients obtained by deleting the i th data point and the other set of regression coefficients obtained by using the whole data set. One way to think of this distance is to plot the coefficients against each other and then measure the distance between the two points that the coefficients generate. Therefore, when CD_i is a large number, the i th data point is having a major effect on the regression coefficients.

Judging from the equation it follows that CD_i depends on three quantities. First, CD_i is effected by the number of independent variables, k . That is, as the number of independent variables increases (other things being held constant) CD_i decreases. Second, CD_i is determined

by the internally studentized residual, which is primarily a measure of the point's relative distance from the center of the data in the y direction. Third, CD_i is adjusted by the leverage of the i th data point, which is a measure of the point's relative distance from the center of the data in the x direction. Hence, a large value of CD_i could be a result of the data point having a large studentized residual, or a large leverage, or both. CD_i values exceeding 1 are considered to be worth further investigation while values exceeding 4 are considered serious outliers (Fox, 1984).

Concrete Examples of Outlier Detection Using Fictitious Data Sets

Four data sets were analyzed using SPSS/PC. The first data set consisted of 12 pairs of X and Y values. The other three sets were created by simply adding one additional data point to the first data set so that $n = 13$. The added point was varied in terms of its "outlyingness" in either the x or y directions. Outlier detection methods of standardized residuals, leverage, externally studentized residuals, and Cook's distance were carried out for each data set. Table 1 lists the regression statistics generated by each data set and Tables 2 through 5 contain the computed values for the various detection techniques.

INSERT TABLE 1 ABOUT HERE

The data set represented in the residual scattergram in Figure 3 and in Table 2 was created to illustrate a least squares linear regression analysis in which the underlying assumptions of regression are met and no outliers are detected. Further, as noted in Table 1 this data set (i.e., X:Y1) generated a multiple R of .68, suggesting a fairly good fit to the linear relationship outlined by the regression parameters.

INSERT FIGURE 3 ABOUT HERE

As can be seen in Figure 1 the residuals appear to be normally distributed about a mean of 0 and their spread about the line $e = 0$ for all values of X appears to be consistent. That is, the residual scores appear to be homoscedastic. Further, none of the diagnostic quantities reveal the presence of an outlier. None of the standardized or studentized residuals are above 2, none of the leverage values exceed the critical value of .33, and none of the CD_i values are above 1. Hence, the regression equation for this data set appears to describe the relationship between the variables well.

INSERT TABLE 2 ABOUT HERE

To create the $X Y_2$ data set a thirteenth data point was added that represented an outlier in y direction. As can be seen in Table 1, multiple R went from .68 to .57, suggesting that the data point is quite influential in the regression analysis. Notably the slope parameter did not change much while the intercept moved upward approximately 1.2 units. This is to be expected as the outlier is towards the center of the X values but lies considerably higher than the center of the Y values. Thus, the line would not be "leveraged" or tilted, but rather, is pulled upward relatively evenly towards the outlier. As can be seen in Table 3, the data point is identified as an outlier only by the standardized and studentized residual. Further, the relative value of the two residual scores illustrates that the studentized residual tends to be more sensitive to outliers in the y direction. Moreover, the studentized residual score of 3.3 marks this point as an extreme outlier while the standardized residual score of 2.29 flags the point as a moderate outlier.

INSERT TABLE 3 ABOUT HERE

The additional data point added to data set X.Y3 is an outlier chiefly in the x direction. Table 4 shows that the only diagnostic value that detects data point 13 as an outlier is the leverage value (i.e., h_i exceeds the critical value of .37). Although the point falls a considerable distance away from the center of the X data, it does not appear to substantially effect the regression analysis. In fact, the multiple R remains virtually unchanged, as noted in Table 1. The addition of this data point then strengthens our confidence in the linear relationship outlined by the regression analysis. This data set is a good example of a data point being flagged as a leverage point but not as an influential outlier. Had the point fallen further off the initial regression line (i.e., prior to adding point 13), in either the negative or positive direction in terms of Y, the regression equation could have been greatly effected.

INSERT TABLE 4 ABOUT HERE

The final data set X.Y4 was created by adding a data point that was an outlier in both directions. As can be seen in Table 1, the regression statistics were all greatly effected by the addition of this one point. Most notably, multiple R went from .69 to .45, considerably lowering our confidence in the fit of the regression line to the data. Data point 13 was flagged as an outlier by the studentized residual score of -2.4 and a Cook's distance value of 1.05. This underscores the point that if only standardized residuals and leverage were used this information would be missed. In fact, in none of the data sets did all four techniques pick up the same point. These data sets have illustrated the need for multiple approaches when attempting to identify influential data points as one approach is not capable of flagging all types of potential outliers.

INSERT TABLE 5 ABOUT HERE

Conclusions

A final note should be made regarding the examples given so far. Data sets presented in this paper are small and contain only one independent and dependent variable. Both these aspects greatly simplify the processes of identifying influential outliers. As more variables are added, multiple dimensions are also added to the data picture. It becomes increasingly difficult to interpret data in multiple dimensions, and so, it becomes increasingly difficult to define influential points in the regression analysis as more and more variables are added. Additionally, the techniques illustrated are more likely to be helpful when there are only one or two outliers. However, it is quite possible that a group of points is exerting influence thus requiring more complicated computations and, at times, different techniques for detection. For example, when attempting to detect multiple outliers by looking at the effect of deleting one point at a time, the number of all possible subsets might be gigantic leading to extensive computations.

The present paper has outlined several techniques for detecting outliers. The underlying premise for using these techniques is that the researcher needs to make sure conclusions based on the data are not solely dependent on one or two extreme observations. If no outliers are detected, then obviously confidence in the findings is bolstered. However, once an outlier is detected the researcher must examine the data point's source of aberration. Outliers can be present in a sample because of errors in recording observations and/or errors in transcription (i.e., measurement error). If this is the case then deleting the observation from the data set is defensible. However, outliers may also be present due to an exceptional occurrence in the investigated phenomenon. Moreover, such an occurrence suggests that the model (i.e., in this

case the least squares regression model) does not adequately describe the relationships among the variables. Only through careful study of the data and the literature can decisions be made as to how to proceed with the analysis when a model violation has occurred. Therefore, outlier detection methods are useful but no substitute for a sound understanding of the phenomenon being studied.

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Table 1

Regression statistics generated by each heuristic data set.

Regression Statistics	X.Y1	X.Y2	X.Y3	X.Y4
Multiple R	.68	.57	.69	.45
R squared	.47	.31	.47	.20
b (slope parameter)	.72	.74	.55	.41
a (intercept parameter)	6.43	7.24	8.20	9.64

Table 2

Diagnostic statistics for data set X:Y1.

	x	y1	estand	lever	estud	cook
1	5.00	12.00	.47649	.19036	.53890	.05890
2	6.50	6.00	-1.22982	.12183	-1.45431	.24558
3	7.00	13.00	.37085	.10237	.39321	.01926
4	9.50	18.00	1.14274	.03046	1.24714	.09460
5	10.00	11.50	-.51075	.02115	-.51965	.01699
6	12.00	9.50	-1.33953	.00085	-1.48089	.09005
7	12.50	17.00	.38166	.00000	.38122	.00722
8	14.00	22.50	1.44741	.00761	1.64174	.11528
9	15.50	16.00	-.37942	.03046	-.38550	.01043
10	17.50	15.00	-.96715	.08460	-1.06766	.11345
11	19.50	25.00	1.09665	.16582	1.31014	.26575
12	21.00	19.50	-.48913	.24450	-.57634	.08680
13	.	.	.	.	.	.

(* Critical leverage value is .33)

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Table 3
Diagnostic statistics for data set X.Y2.

	x	y2	estand	lever	estud	cook
1	5.00	12.00	.18386	.19216	.20548	.00851
2	6.50	6.00	-1.06156	.12330	-1.21207	.17637
3	7.00	13.00	.09934	.10372	.10470	.00133
4	9.50	18.00	.65038	.03122	.67126	.02875
5	10.00	11.50	-.55278	.02179	-.56392	.01857
6	12.00	9.50	-1.16264	.00098	-1.23999	.06192
7	12.50	17.00	.08582	.00001	.08519	.00033
8	14.00	22.50	.85423	.00722	.88367	.03660
9	15.50	16.00	-.47875	.02966	-.48866	.01530
10	17.50	15.00	-.91350	.08324	-.99649	.09475
11	19.50	25.00	.57802	.16388	.64556	.06979
12	21.00	19.50	-.57983	.24210	-.68551	.11565
13	13.00	30.00	2.29741	.00072	3.29283	.24085

(*Critical leverage value is .37)

Table 4
Diagnostic statistics for data set X:Y3

	x	y3	estand	lever	estud	cook
1	5.00	12.00	.24360	.14063	.26349	.01054
2	6.50	6.00	-1.40679	.09664	-1.66817	.25145
3	7.00	13.00	.21557	.08380	.22493	.00530
4	9.50	18.00	1.08574	.03333	1.17021	.08209
5	10.00	11.50	-.55064	.02598	-.56303	.01939
6	12.00	9.50	-1.30284	.00572	-1.42207	.08335
7	12.50	17.00	.44022	.00295	.44182	.00914
8	14.00	22.50	1.56579	.00010	1.78426	.11083
9	15.50	16.00	-.20530	.00548	-.20477	.00206
10	17.50	15.00	-.71611	.02545	-.74015	.03258
11	19.50	25.00	1.42836	.06006	1.65452	.18762
12	21.00	19.50	-.10135	.09562	-.10629	.00129
13	29.00	21.50	-.69625	.42424	-.98441	.48816

(*Critical leverage value is .37)

Table 5
Diagnostic statistics for data set X:Y4.

	x	y4	estand	lever	estud	cook
1	5.00	12.00	.06401	.16836	.07027	.00088
2	6.50	6.00	-1.27317	.11351	-1.49174	.23549
3	7.00	13.00	.10068	.09762	.10571	.00130
4	9.50	18.00	.90467	.03614	.95691	.05881
5	10.00	11.50	-.45085	.02743	-.45898	.01322
6	12.00	9.50	-1.02071	.00459	-1.07226	.05033
7	12.50	17.00	.45423	.00187	.45590	.00958
8	14.00	22.50	1.44206	.00091	1.60583	.09516
9	15.50	16.00	.00379	.01072	.00378	.00000
10	17.50	15.00	-.36389	.04056	-.37188	.00999
11	19.50	25.00	1.49235	.08956	1.79119	.26684
12	21.00	19.50	.25625	.13888	.27696	.01152
13	24.00	11.50	-1.60941	.26986	-2.37419	1.05255

(*Critical leverage value is .37)

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Figure 1
Scatterplots of residual scores against the independent variable.

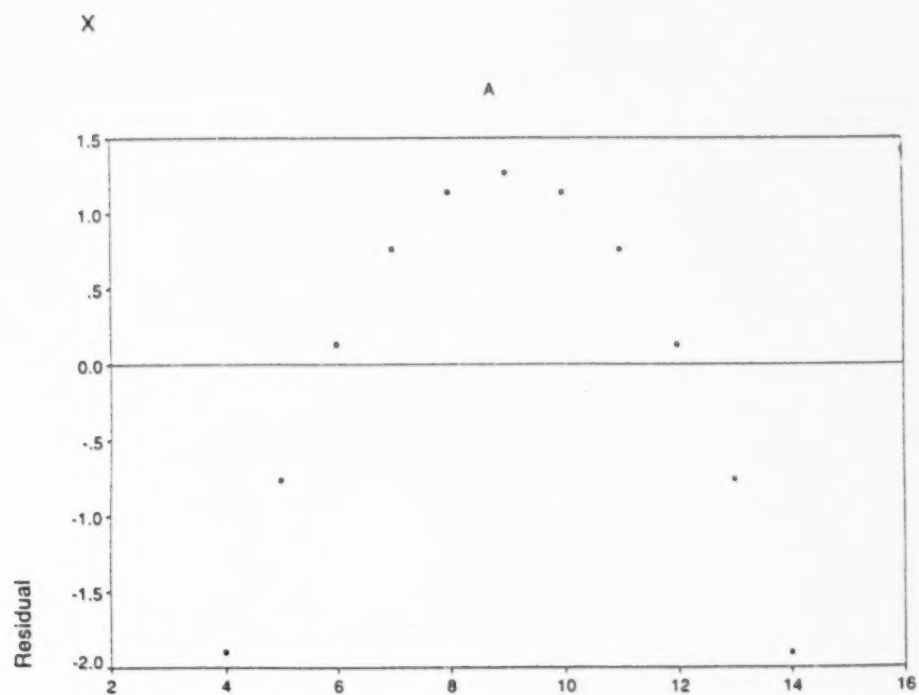
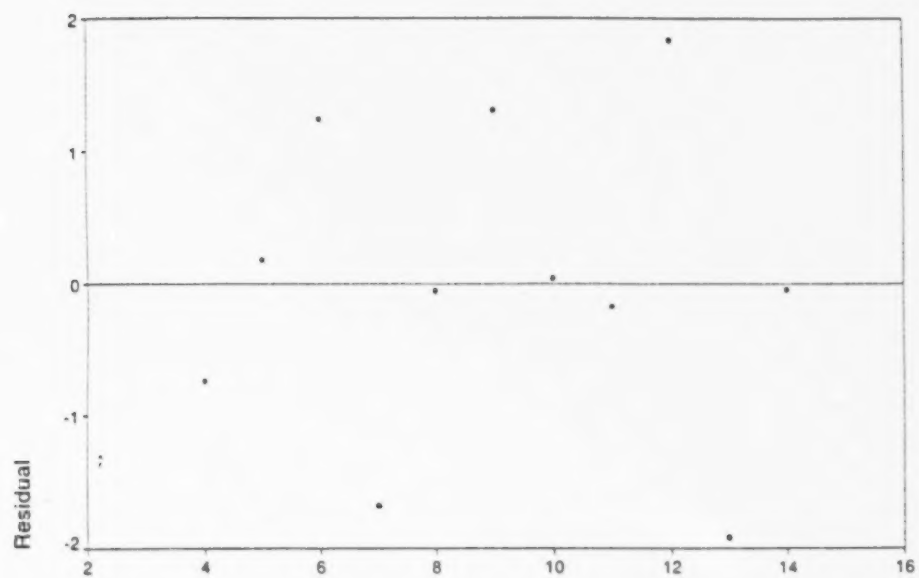
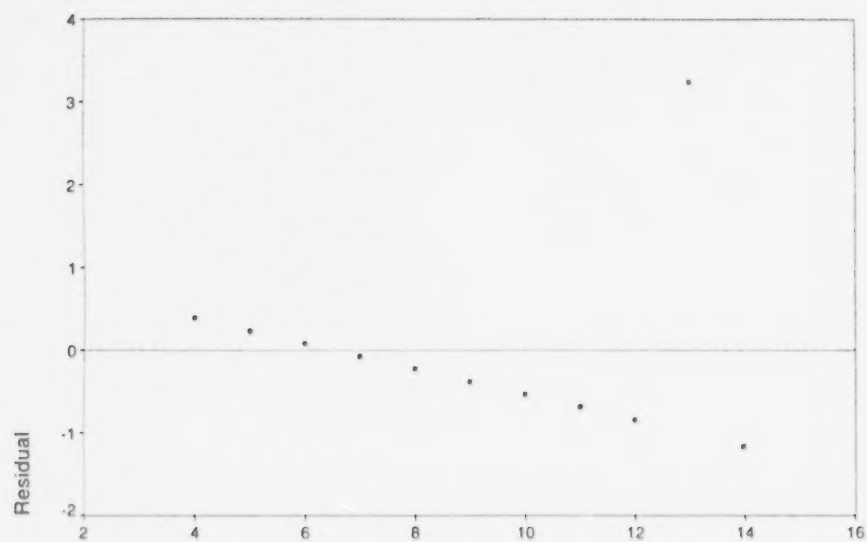
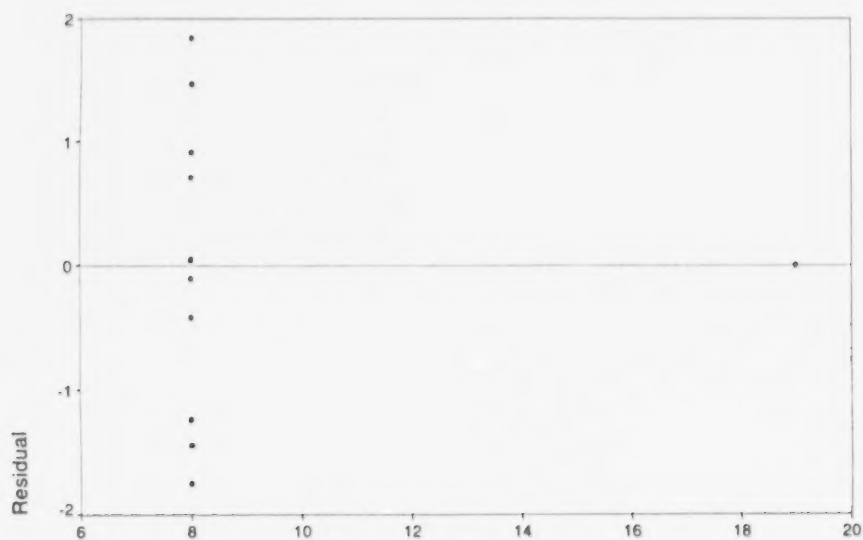


Figure 1
Scatterplots of residual scores against the independent variable.



X

C



X

D

0.14
0.08

Figure 2

Scatterplot of independent variable against dependent variable for Figure 1D data.

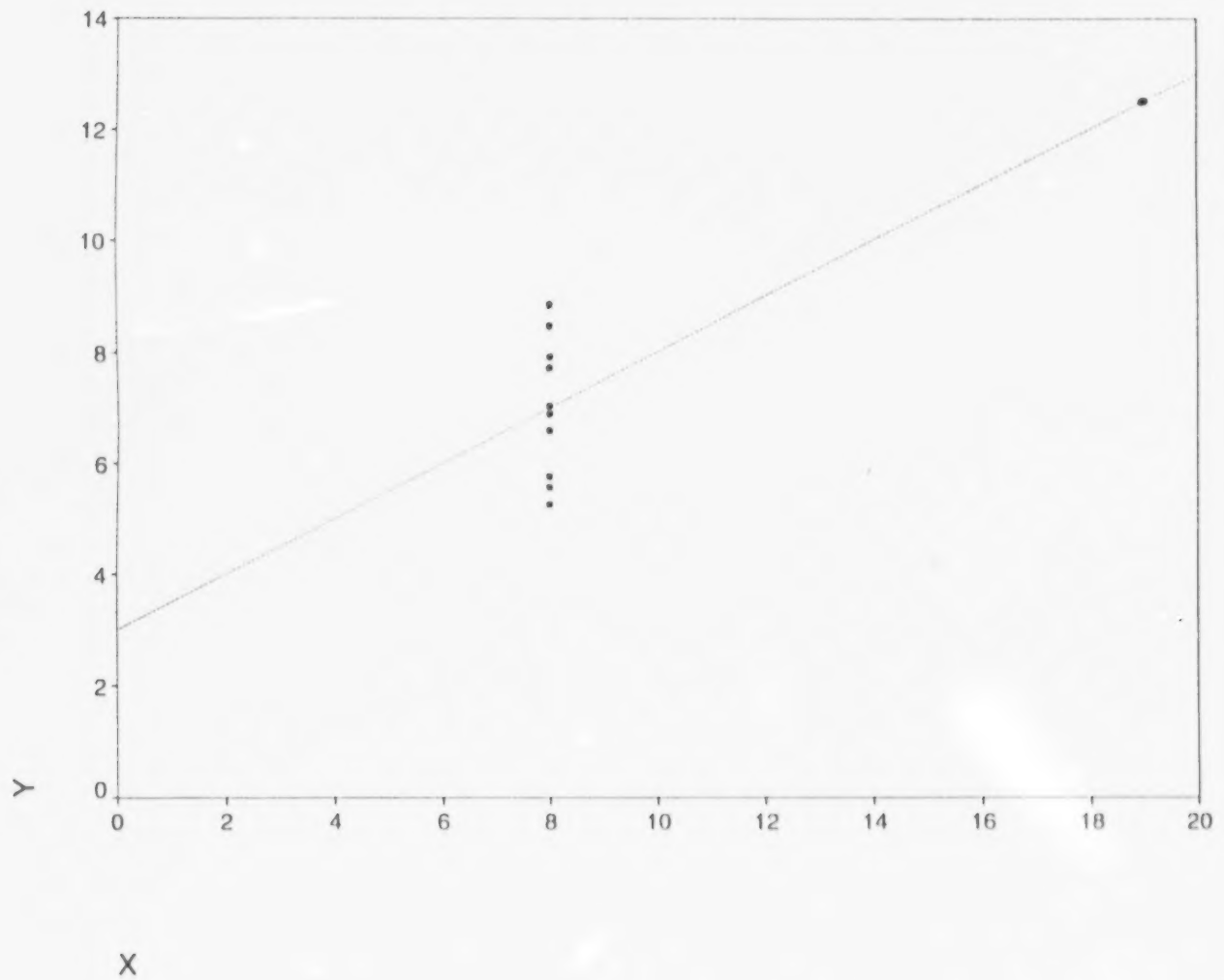


Figure 3
Residual scatterplot for data set X:Y1.

